Simplifying with variables
CHAPTER 2  Simplifying with Variables

This chapter begins with a focus on the use of variables, such as \( x \) and \( y \). First, you will use tools called “algebra tiles” to explore how and where to use variables. You will write expressions and then learn how to write them in the most efficient way.

Later in the chapter, you will learn how to compare two expressions to determine which one is greater. This will lead to a process of recording your steps so that anyone can understand your work. Finally, you will look at what happens when you cannot tell which expression is greater and consider the question, “What if they are equal?”

Since the topics in this chapter lay the foundation for simplifying expressions and solving equations, they will be revisited and built upon throughout the course.

In this chapter, you will learn:

- What a variable is.
- How to write and simplify algebraic expressions.
- How to compare two complicated algebraic expressions.
- How to solve for a variable if you know that two expressions are equal.

Guiding Questions

Think about these questions throughout this chapter:

What is a variable?
What can I do with a variable?
How can I solve for a variable?
How many different ways can I write an expression?

Chapter Outline

Section 2.1 This section, the only section of the chapter, introduces algebra tiles. Using algebra tiles will help develop the symbolic manipulation skills of combining like terms and solving linear equations. A special focus will be placed on the meaning of “minus” and how to make “zero.”
### 2.1.1 What is a variable?

Exploring Variables and Expressions

In Algebra and in future mathematics courses, you will work with unknown quantities that can be represented using variables. Today manipulatives called “algebra tiles” will be introduced to help you and your teammates answer some important questions, such as “What is a variable?” and “How can we use it?”

2-1. Your teacher will distribute a set of algebra tiles for your team to use during this course. As you explore the tiles, address the following questions with your team. Be prepared to share your responses with the class.

- How many different shapes are there? What are all of the different shapes?
- How are the shapes different? How are they the same?
- How are the shapes related? Which fit together and which do not?

2-2. Draw a picture of each type of tile on your paper.

a. The algebra tiles will be referred to by their areas. Since the smallest square has a length of 1 unit, its area is 1 square unit. Thus, the name for this tile is “one” or a “unit tile.” Label your drawing of this tile with its side lengths and area.

b. Can you use the unit tile to find the other lengths? Why or why not?

c. If you cannot use the unit tile to measure all of the other side lengths of the other tiles, how can you label them so that you can find their area? Discuss this with your team and be prepared to share your ideas with the class.

d. Label your drawings and then name the other tiles using their areas. Be sure to use what you know about the area of a rectangle and the area of a square.
2-3. JUMBLED PILES

Your teacher will show you a jumbled pile of algebra tiles and will challenge you to write a name for the collection. What is the best description for the collection of tiles? Is your description the best possible?

2-4. Build each collection of tiles represented below. Then name the collection using a simpler algebraic expression, if possible. If it is not possible to simplify the expression, explain why not.

a. \(3x + 5 + x^2 + y + 3x^2 + 2\)  
b. \(2x^2 + 1 + xy + x^2 + 2xy + 5\)

c. \(2 + x^2 + 3x + y^2 + 4y + xy\)  
d. \(3y + 2 + 2xy + 4x + y^2 + 4y + 1\)

2-5. LEARNING LOG

In your Learning Log, explain what a variable is in your own words. Describe each type of tile with a diagram that includes each dimension and an area label. Explain when tiles can and cannot be combined. Be sure to include examples to support your statements. Title this entry “Algebra Tiles and Variables” and include today’s date.
2-6. Suppose you put one of your $x$-tiles and two unit tiles with another pile of three $x$-tiles and five unit tiles. What is in this new pile? Write it as a sum.

2-7. Suppose one person in your team has two $x^2$-tiles, three $x$-tiles, and one unit tile on his desk and another person has one $x^2$-tile, five $x$-tiles, and eight unit tiles on her desk. You decide to put all of the tiles together on one desk. What is the name for this new group of tiles?

2-8. Copy the following figures onto your paper. Then find the area and perimeter of each shape. Assume that all corners are right angles. Show all work.

\[ \begin{array}{c}
\text{a.} \\
\begin{array}{c}
\begin{array}{c}
25 \\
16 \\
12 \\
19 \\
13 \\
35 \\
\end{array}
\end{array}
\end{array} \]

\[ \begin{array}{c}
\text{b.} \\
\begin{array}{c}
\begin{array}{c}
8 \\
12 \\
7 \\
8 \\
8 \\
12 \\
\end{array}
\end{array}
\end{array} \]

2-9. One meaning of the word evaluate is to find the value of an expression for a specified value of the variable(s). To evaluate, replace a variable with a number and calculate the result. For example, when you are asked to evaluate the expression $4x - 2$ when $x = -7$, you would put $-7$ in place of the variable and calculate: $4 \cdot (-7) - 2 = -30$.

Evaluate the expressions below for the given values of $x$ and $y$.

a. $\frac{6}{x} + 9$ if $x = 3$

b. $8x - 3 + y$ if $x = 2$ and $y = 1$

c. $2xy$ if $x = 5$ and $y = -3$

d. $2x^2 - y$ if $x = 3$ and $y = 8$

2-10. For the following problem, define a variable and write an equation (use the 5-D Process if needed). Then solve the equation to solve the problem. Write your solution as a sentence.

A cable 84 meters long is cut into two pieces so that one piece is 18 meters longer than the other. Find the length of each piece of cable.
2.1.2 What is the perimeter?

Simplifying Expressions by Combining Like Terms

While Lesson 2.1.1 focused on the area of algebra tiles, today’s lesson will focus on the perimeter. What is perimeter? How can you find it? By the end of this lesson, you will be able to find the perimeter of complex shapes formed by collections of tiles.

Your class will also focus on several ways to find perimeter, recognizing that there are different ways to “see” or recognize perimeter. Sometimes, with complex shapes, a convenient shortcut can help you find the perimeter more quickly. Be sure to share any insight into finding perimeter with your teammates and with the whole class.

While working today, ask yourself and your teammates these focus questions:

- How did you see it?
- How can you write it?
- Is your expression as simplified as possible?

2-11. Your teacher will provide a set of algebra tiles for your team to use today. Separate one of each shape and review its name (area). Then find the perimeter of each tile. Decide with your team how to write a simplified expression that represents the perimeter of each tile. Be prepared to share the perimeters you find with the class.

2-12. Each part of an expression that is separated by addition or subtraction signs is called a term. If two terms contain the same variable(s), including the same exponents, if any, they are called like terms.

Match the terms in the left column with the like terms that match them in the right column.

A. \(3x\)  
B. \(-4y^2\)  
C. \(6a^4\)  
D. \(10s\)  
E. \(-8\)  
F. \(6xy\)  
G. \(c\)  

1. \(6y^2\)  
2. \(7xy\)  
3. \(13\)  
4. \(-7c\)  
5. \(9a^4\)  
6. \(s\)  
7. \(5x\)
2-13. For each of the shapes formed by algebra tiles below:
   - Use tiles to build the shape.
   - Sketch and label the shape on your paper. Then write an expression that represents the perimeter.
   - Simplify your perimeter expression as much as possible. This process of writing the expression more simply by collecting together the parts of the expression that are the same is called **combining like terms**.

   a.  
   
   b.  

   c.  
   
   d.  

2-14. Calculate the perimeter of the shapes in problem 2-13 if the length of each $x$-tile is 3 units and the length of each $y$-tile is 8 units. Show all work.

2-15. EXTENSION

The perimeter of the shape at the right is 32 units. Find possible values for $x$ and $y$. Is there more than one possible solution for each variable? If so, find another solution. If not, explain how you know there is only one solution.

2-16. LEARNING LOG

In your Learning Log, create your own shape using three different-shaped tiles. Draw the shape and show how to write a simplified expression for its perimeter. Label this entry “Finding Perimeter and Combining Like Terms” and include today’s date.
**Variable:** A letter or symbol that represents one or more numbers.

**Expression:** A combination of numbers, variables, and operation symbols. For example, $2x + 3(5 - 2x) + 8$. Also, $5 - 2x$ is a smaller expression within the larger expression.

**Term:** Parts of the expression separated by addition and subtraction. For example, in the expression $2x + 3(5 - 2x) + 8$, the three terms are $2x$, $3(5 - 2x)$, and $8$. The expression $5 - 2x$ has two terms, 5 and $-2x$.

**Coefficient:** The numerical part of a term. In the expression $2x + 3(5 - 2x) + 8$, for example, 2 is the coefficient of $2x$. In the expression $7x - 15x^2$, both 7 and 15 are coefficients.

**Constant term:** A number that is not multiplied by a variable. In the expression $2x + 3(5 - 2x) + 8$, the number 8 is a constant term. The number 3 is not a constant term, because it is multiplied by a variable inside the parentheses.

**Factor:** Part of a multiplication expression. In the expression $3(5 - 2x)$, 3 and $5 - 2x$ are factors.
2-17. Simplify each algebraic expression below, if possible. If it is not possible to simplify the expression, explain why not.

a. \(3y + 2y + y^2 + 5 + y\)  
b. \(3y^2 + 2xy + 1 + 3x + y + 2x^2\)

c. \(3xy + 5x + 2 + 3y + x + 4\)  
d. \(4m + 2mn + m^2 + m + 3m^2\)

2-18. If the tiles have the dimensions shown at right, what is the name of the tile collection below? (That is, what is the total area of all of the pieces?) Write the expression algebraically, using \(x\), \(x^2\), \(y\), \(y^2\), and \(xy\).

![Diagram of tiles]

2-19. Remember that one meaning of the word “evaluate” is to replace a variable with a number and to calculate the result. For example, evaluating the expression \(x^2\) when \(x = -2\) results in the solution \((-2)^2 = 4\).

Evaluate the expressions below for the given values.

a. \(-4d + 3\) if \(d = -1\)  
b. \(k - m\) if \(k = 4\) and \(m = -10\)

c. \(\frac{t}{w}\) if \(t = 6\) and \(w = -3\)  
d. \(x^2 + y^2\) if \(x = 7\) and \(y = 5\)
2-20. Solve the following problems.

   a. A typical small bag of colored candies has about 135 candies in it, 27 of which are blue. At this rate, how many blue candies would you expect in a pile of 1000 colored candies?

   b. Ten calculators cost $149.50. How much would 100 cost? 1000? 500?

2-21. Examine the graph at right.

   a. Name the coordinates of points A, B, C, and D in (x, y) form.

   b. Draw a set of axes like the ones shown at right on graph paper. Then plot points E(5, 2), F(−3, −1), G(0, −4), and H(2, −3).
2.1.3 What does “minus” mean?

Writing Algebraic Expressions

In this section, you will look at algebraic expressions and see how they can be interpreted using an Expression Mat. To achieve this goal, you first need to understand the different meanings of the “minus” symbol, which is found in expressions such as $5 - 2$, $-x$, and $-(-3)$.

2-22. LEARNING LOG

What does “−” mean? Find as many ways as you can to describe this symbol and discuss how these descriptions differ from one another. Share your ideas with the class and record the different uses in your Learning Log. Title this entry “Meanings of Minus” and include today’s date.

2-23. USING AN EXPRESSION MAT

Your introduction to algebra tiles in Lessons 2.1.1 and 2.1.2 involved only positive values. Today you will look at how you can use algebra tiles to represent “minus.” Below are several tiles with their associated values. Note that the shaded tiles are positive and the un-shaded tiles are negative. The diagram at right will appear throughout the text as a reminder.

\[
\begin{align*}
\begin{array}{c}
\text{□ □ □} = 5 \\
\text{□ □} = -3 \\
\text{□ □ □ □} = 3x \\
\text{□ □ □ □ □ □} = -2y
\end{array}
\end{align*}
\]

“Minus” can also be represented with a new tool called an Expression Mat, shown at right. An Expression Mat is an organizing tool that will be used to represent expressions. Notice that there is a positive region at the top and a negative (or “opposite”) region at the bottom.
Using the Expression Mat, the value \(-3\) can be shown in several ways, two of which are shown at right.

Note that in these examples, the diagram on the left side uses negative tiles in the "+" region, while the diagram on the right side uses positive tiles in the "−" region.

a. Build two different representations for \(-2x\) using an Expression Mat.

b. Similarly, build \(3x − (−4)\). How many different ways can you build \(3x − (−4)\)?

2-24. As you solved problem 2-23, did you see all of the different ways to represent "minus" that you listed in problem 2-22? Discuss how you could use an Expression Mat to represent the different meanings discussed in class.

2-25. BUILDING EXPRESSIONS

Use the Expression Mat to create each of the following expressions with algebra tiles. Find at least two different representations for each expression. Sketch each representation on your paper. Be prepared to share your different representations with the class.

a. \(-3x + 4\)  b. \(-(y − 2)\)  c. \(-y − 3\)  d. \(5x − (3 − 2x)\)

2-26. In problem 2-25, you represented algebraic expressions with algebra tiles. In this problem, you will need to reverse your thinking to write an expression from a diagram of algebra tiles.

Working with a partner, write algebraic expressions for each representation below. Start by building each problem using your algebra tiles.

a.  

b.  

c.  

d.  

2-27. Patti, Emilie, and Carla are debating the answer to part (d) of problem 2-26. Patti wrote $2 - 1 + 2x - 3$. Carla thinks that the answer is $2x + 2 - 4$. Emilie is convinced that the answer is $2x - 2$. Discuss with your team how each person might have arrived at her answer. Who do you think is correct? When you decide, write an explanation on your paper and justify your answer.

2-28. LEARNING LOG

Reflect about what you have learned from today’s lesson as you answer the following question in your Learning Log. Title this entry “Representing Expressions on an Expression Mat” and include today’s date.

Using an Expression Mat, find two different ways to represent $x - 1 - (2x - 3)$. Sketch the different representations and write a few sentences to describe the differences in the ways you built each representation.

METHODS AND MEANINGS

**Combining Like Terms**

Combining tiles that have the same area to write a simpler expression is called combining like terms. See the example shown at right.

$\text{Example 1: } 2x^2 + xy + y^2 + x + 3 + x^2 + 3xy + 2 \quad \Rightarrow \quad 3x^2 + 4xy + y^2 + x + 5$

$\text{Example 2: } 3x^2 - 2x + 7 - 5x^2 + 3x - 2 \quad \Rightarrow \quad -2x^2 + x + 5$

A term is an algebraic expression that is a single number, a single variable, or the product of numbers and variables. The simplified algebraic expression in Example 2 above contains three terms. The first term is $-2x^2$, the second term is $x$, and the third term is 5.
2-29. Copy and write the following expressions by combining like terms. Using or drawing sketches of algebra tiles may be helpful.

a. \(2x + 3x + 3 + 4x^2 + 10 + x\)  
b. \(4x + 4y^2 + y^2 + 9 + 10 + x + 3x\)

c. \(2x^2 + 30 + 3x^2 + 4x^2 + 14 + x\)  
d. \(20 + 5xy + 4y^2 + 10 + y^2 + xy\)

2-30. Find a simplified algebraic expression for each Expression Mat below.

a.  

b.  

c.  

2-31. Write an equation and solve it to find the answer to the question below. Use the 5-D Process to help you write the equation, if needed. Remember to define your variable and to write your answer in a complete sentence.

Susan is buying three different colors of tiles for her kitchen floor. She is buying 25 more red tiles than beige tiles, and three times as many navy-blue tiles as beige tiles. If Susan buys 435 tiles altogether, how many tiles of each color does she buy?

2-32. Without using a calculator, compute the value of each expression below.

a. \(-14 + (-31)\)  
b. \(-(8) - (-2)\)

c. \(-\frac{16}{8}\)  
d. \(-11 \cdot 24\)

e. \(\frac{1}{2} - \frac{3}{4}\)  
f. \(46 ÷ (-23)\)
2-33. To bake 100 of his favorite cookies, Mr. Wallis needs 350 grams of sugar.

<table>
<thead>
<tr>
<th>Cookies</th>
<th>Sugar (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>350</td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>700</td>
</tr>
<tr>
<td></td>
<td>1400</td>
</tr>
</tbody>
</table>

a. How many grams of sugar would he need to bake 10 cookies? What about 20 cookies? Show all work.

b. What is the unit rate? That is, how many grams of sugar are needed for 1 cookie?

c. To help him know how much sugar to use when baking cookies, Mr. Wallis started to make a table, as shown above. Copy and complete his table on your paper.
2.1.4 What makes zero?

Using Zero to Simplify Algebraic Expressions

Today you will continue your work with rewriting algebraic expressions. As you work with your team, ask yourself and your teammates these focus questions:

How did you see it?
How can you write it?
Is your expression as simplified as possible?

2-34. LIKELY STORY!

Imagine the following situations:

- Julie baby-sits for her neighbor's baby and stuffs the $15 she earned into her purse. When she gets home, the $15 is nowhere to be found. It must have fallen out of her purse.

- The Burton Pumas football team completes a pass and gains 12 yards. But on the very next play, the quarterback holds onto the ball too long and gets sacked, losing 12 yards.

- Rolando is at the beach. He digs a hole in the sand and places the sand he removes in a pile next to his hole. Someone comes along and pushes the pile back into the hole.

What do each of these situations have in common? Can you represent each of them using symbols? How?

2-35. How can you represent zero with tiles on an Expression Mat? With your team, try to find at least two different ways to do this (and more if you can). Be ready to share your ideas with the class.
2-36. Gretchen used seven algebra tiles to build the expression shown below.

a. Build this collection of tiles on your own Expression Mat and write its value.

b. Represent this same value three different ways, each time using a different number of tiles. Be ready to share your representations with the class.

2-37. Build each expression below so that your representation does not match those of your teammates. Once your team is convinced that together you have found four different, valid representations, sketch your representation on your paper and be ready to share your answer with the class.

a. \(-3x + 5 + y\)  
b. \(-(-2y + 1)\)  
c. \(2x - (x - 4)\)

2-38. Write the algebraic expression shown on each Expression Mat below. Build the model and then simplify the expression by removing as many tiles as you can without changing the value of the expression. Finally, write the simplified algebraic expression.

a.  

b.  

2-39. Simplify each of the following expressions by building it on your Expression Mat and removing zeros. Your teacher will give you instructions about how to represent your work on your paper.

a. \(3x - (2x + 4)\)  
b. \(7 - (4y - 3) + 2y - 4\)

2-40. LEARNING LOG

In your Learning Log, describe the different ways you can represent zero using your Expression Mat. Include an example and be sure to draw the tiles. Title this entry “Using Zeros to Simplify” and include today’s date.
METHODS AND MEANINGS

Evaluating Expressions and the Order of Operations

To **evaluate** an algebraic expression for particular values of the variables, replace the variables in the expression with their known numerical values and simplify. Replacing variables with their known values is called **substitution**. An example is provided below.

Evaluate \(4x - 3y + 7\) for \(x = 2\) and \(y = 1\).

Replace \(x\) and \(y\) with their known values of 2 and 1, respectively, and simplify.

\[
4(2) - 3(1) + 7 = 8 - 3 + 7 = 12
\]

When evaluating a complex expression, you must remember to use the **Order of Operations** that mathematicians have agreed upon. As illustrated in the example below, the order of operations is:

Original expression: \((10 - 3 \cdot 2) \cdot 2^2 - \frac{13 - 3^2}{2} + 6\)

Circle expressions that are grouped within parentheses or by a fraction bar:

Simplify within circled terms using the order of operations:

- Evaluate exponents.

- Multiply and divide from left to right.

- Combine terms by adding and subtracting from left to right.

Circle the remaining terms:

Simplify within circled terms using the Order of Operations as described above.

\[
16 - 2 + 6 = 20
\]
2-41. Can zero be represented by any number of tiles? Using only the unit tiles (in other words, only the 1 and -1 tiles), determine whether you can represent zero on an Expression Mat with the number of tiles below. If you can, draw an Expression Mat demonstrating that it is possible. If it is not possible, explain why not.

a. 2 tiles  

b. 6 tiles  

c. 3 tiles

2-42. Write and simplify the algebraic expression shown in each Expression Mat below.

a. 

b. 

c. 

2-43. Read the Math Notes box for this lesson. Then evaluate each expression below.

a. For \( y = 2 + 3x \) when \( x = 4 \), what does \( y \) equal?

b. For \( a = 4 - 5c \) when \( c = -\frac{1}{2} \), what does \( a \) equal?

c. For \( n = 3d^2 - 1 \) when \( d = -5 \), what does \( n \) equal?

d. For \( v = -4(r - 2) \) when \( r = -1 \), what does \( v \) equal?

e. For \( 3 + k = t \) when \( t = 14 \), what does \( k \) equal?

2-44. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.

a. 

b. 

c. 

d. 
2-45. Find the perimeter of the entire rectangle shown at right (that is, the length of the outside boundary of the figure). Notice that the areas of two of the parts have been labeled inside the rectangle. Also find the total area. Remember to show all work leading to your solution.
2.1.5 How can I simplify the expression?

Using Algebra Tiles to Simplify Algebraic Expressions

Which is greater: 58 or 62? That question might seem easy, because the numbers are ready to be compared. However, if you are asked which is greater, \(2x + 8 - x - 3\) or \(6 + x + 1\), the answer is not so obvious! In this lesson, you and your teammates will investigate how to compare two algebraic expressions and decide whether one is greater.

2-46. For each expression below:

- Use an Expression Mat to build the expression.
- Find a different way to represent the same expression using tiles.

a. \(7x - 3\)  

b. \(-(-2x + 6) + 3x\)

2-47. COMPARING EXPRESSIONS

Two expressions can be represented at the same time using an **Expression Comparison Mat**. The Expression Comparison Mat puts two Expression Mats side-by-side so you can compare them and see which one is greater. For example, in the picture at right, the expression on the left represents \(-3\), while the expression on the right represents \(-2\). Since \(-2 > -3\), the expression on the right is greater.

Build the Expression Comparison Mat shown at right. Write an expression representing each side of the Expression Mat.

a. Can you simplify each of the expressions so that fewer tiles are used? Develop a method to simplify both sides of the Expression Comparison Mats. Why does it work? Be prepared to justify your method to the class.

b. Which side of the Expression Comparison Mat do you think is greater (has the largest value)? Agree on an answer as a team. Make sure each person in your team is ready to justify your conclusion to the class.
2-48. As Karl simplified some algebraic expressions, he recorded his work on the diagrams below.

- Explain in writing what he did to each Expression Comparison Mat on the left to get the Expression Comparison Mat on the right.
- If necessary, simplify further to determine which Expression Mat is greater. How can you tell if your final answer is correct?

a. 

b. 

c. 

2-49. Use Karl's "legal" simplification moves to determine which side of each Expression Comparison Mat below is greater. Record each of your "legal" moves on the Lesson 2.1.5A Resource Page by drawing on it the way Karl did in problem 2-49. After each expression is simplified, state which side is greater (has the largest value). Be prepared to share your process and reasoning with the class.

a. [Diagram showing a left side with a plus sign and a right side with a plus sign.]

b. [Diagram showing a left side with a plus sign and a right side with an X sign.]

2-50. LEARNING LOG

In your Learning Log, explain each of the types of "legal" moves that you can use to simplify and compare expressions. For each type of "legal" move, sketch an example. Title this entry "Legal Moves for Simplifying and Comparing Expressions" and include today's date.

METHODS AND MEANINGS

MATH NOTES

Commutative Properties

The Commutative Property of Addition states that when adding two or more number or terms together, order is not important. That is:

\[ a + b = b + a \]

For example, \( 2 + 7 = 7 + 2 \)

The Commutative Property of Multiplication states that when multiplying two or more numbers or terms together, order is not important. That is:

\[ a \cdot b = b \cdot a \]

For example, \( 3 \cdot 5 = 5 \cdot 3 \)

However, subtraction and division are not commutative, as shown below:

\[ 7 - 2 \neq 2 - 7 \text{ since } 5 \neq -5 \]

\[ 50 \div 10 \neq 10 \div 50 \text{ since } 5 \neq 0.2 \]
2-51. Simplify the following expressions by combining like terms, if possible.
   a. \(x + x - 3 + 4x^2 + 2x - x\)  
   b. \(8x^2 + 3x - 13x^2 + 10x^2 - 25x - x\)
   c. \(4x + 3y\)  
   d. \(20 + 3xy - 3 + 4y^2 + 10 - 2y^2\)

2-52. When writing an expression for part (a) of problem 2-42, Ricardo wrote \(2x - 3 - (x + 1)\), while Francine wrote \(-3 + 2x - (x + 1)\). Francine states that their expressions are equivalent. Is Francine’s conclusion true or false? Use algebraic properties to justify your conclusion.

2-53. The two lines at right represent the growing profits of Companies A and B.
   a. Sketch this graph on your paper. If Company A started out with more profit than Company B, determine which line represents A and which represents B. Label the lines appropriately.
   b. In how many years will both companies have the same profit?
   c. Approximately what will that profit be?
   d. Which company’s profits are growing more quickly? How can you tell?

2-54. Evaluate each expression to find \(y\).
   a. \(y = 2 + 4.3x\) when \(x = -6\)  
   b. \(y = (x - 3)^2\) when \(x = 9\)
   c. \(y = x - 2\) when \(x = 3.5\)  
   d. \(y = 5x - 4\) when \(x = -2\)

2-55. When baking cupcakes for his class of 21 students, Sammy needed two eggs. Now he wants to bake cupcakes for the upcoming science fair. If he expects 336 people to attend the science fair, how many eggs will he need?
Lesson 2.1.5A Resource Page
Problem 2-46

a. Left
   +
   
   Which is greater?
   
   
   
   
   Right
   +
   
   Which is greater?
   
   
   
   

b. Left
   +
   
   Which is greater?
   
   
   
   
   Right
   +
   
   Which is greater?
   
   
   
   

[Diagram of squares representing numbers]

[Legend: □ = +1, □ = -1]
Lesson 2.1.5B Resource Page

Expression Comparison Mat

Which is greater?

Left

Right
2.1.6 Which is greater?

Using Algebra Tiles to Compare Expressions

Can you always tell whether one algebraic expression is greater than another? In this lesson, you will compare the values of two expressions, practicing the different simplification strategies you have learned so far.

2-56. WHICH IS GREATER?

Write an algebraic expression for each side of the Expression Comparison Mats given below. Use the “legal” simplification moves you worked with in Lesson 2.1.5 to determine which expression on the Expression Comparison Mat is greater.

a. 

b. 

c. 

d. 

e. 

f. 

2-57. Build the Expression Comparison Mat shown below with algebra tiles.

a. Simplify the expressions using the "legal" moves that you developed in Lesson 2.1.5.

b. Can you tell which expression is greater? Explain in a few sentences on your paper. Be prepared to share your conclusion with the class.

METHODS AND MEANINGS
Simplifying an Expression ("Legal Moves")

Three common ways to simplify or alter expressions on an Expression Mat are illustrated below.

- Removing an equal number of opposite tiles that are in the same region. For example, the positive and negative tiles in the same region at right combine to make zero.

- Flipping a tile to move it out of one region into the opposite region (i.e., finding its opposite). For example, the tiles in the "-" region at right can be flipped into the "+" region.

- Removing an equal number of identical tiles from both the "-" and the "+" regions. This strategy can be seen as a combination of the two methods above, since you could first flip the tiles from one region to another and then remove the opposite pairs.
2-58. WHICH IS GREATER?

For each Expression Comparison Mat below, simplify and determine which side is greater.

a.  

b.  

2-59. Cairo wants to create a graph that represents the heights and bases of all rectangles that have an area of 36 square units. He started by drawing the rectangles A, B, C, and D at right. Examine the dimensions (length and width) of each rectangle.

a. Copy the graph at right onto graph paper. Then match the letter of each rectangle above with a point on the graph. Which point is not matched?

b. What are the base, height, and area for the unmatched point?

c. Why should the unmatched point not be on Cairo’s graph?

d. Find the dimensions of three more rectangles that have areas of 36 square units. At least one of your examples should have dimensions that are not integers. Place a new point on the graph for each new rectangle you find.

e. Connect all of the points representing an area of 36 square units. Describe the resulting graph.
2-60. Use substitution to find $y$.

a. $y = 2 + 4.3x$, when $x = -6$

b. $y = x - 2$, when $x = 3.5$

c. $y = (x - 3)^2$, when $x = 9$

d. $y - 5x = -4$, when $x = -2$

2-61. One of Teddy's jobs at home is to pump gas for his family's sedan and truck. When he fills up the sedan with 12 gallons of gas, he notices that it costs him $50.28.

a. How much does one gallon of gas cost? This is also called the unit rate. Explain how you found your answer.

b. How much will it cost him to fill up the truck if it needs 25 gallons of gas? Show your work.

c. When Teddy filled up the tank on his moped, it cost $5.03. How much gas did his moped need? Explain how you know.

2-62. The graph below shows distances traveled by Car A and Car B. Car A is represented by the line containing point A, and Car B is represented by the line containing point B. Use the graph to answer the following questions.

![Distance vs Time Graph](image)

a. Which car is traveling faster? How can you tell?

b. Find the coordinates of point A and point B.

c. How fast did Car A travel (in miles per hour)? How fast did Car B travel?

d. Does the distance Car A has traveled vary directly with the time? Why or why not?
2.1.7 How can I write it?

Simplifying and Recording Work

Today you will continue to compare expressions as you strengthen your simplification strategies. At the same time, you will work with your class to find ways to record your work so that another student can follow your strategies.

2-63. Use algebra tiles to build the expressions below on an Expression Comparison Mat. Use “legal” simplification moves to determine which expression is greater, if possible. If it is not possible to tell which expression is greater, explain why.

a. Which is greater: \(3x - (2 - x) + 1\) or \(-5 + 4x + 4\)?

b. Which is greater: \(2x^2 - 2x + 6 - (3x)\) or \(-3(2x^2) + 5 + 2x\)?

c. Which is greater: \(-1 + 6y - 2 + 4x - 2y\) or \(x + 5y - (-2 + y) + 3x - 6\)?

2-64. RECORDING YOUR WORK

Although using algebra tiles can make some things easier because you can “see” and “touch” the math, it can be difficult to remember what you did to solve a problem unless you take good notes.

Use the simplification strategies you have learned to determine which expression on the Expression Comparison Mat at right is greater. Record each step as instructed by your teacher. Also record the simplified expression that remains after each move. This will be a written record of how you solved this problem. Discuss with your team what the best way is to record your moves.
2-65. While Athena was comparing the expressions shown at right, she was called out of the classroom. When her teammates needed help, they looked at her paper and saw the work shown below. Unfortunately, she had forgotten to explain her simplification steps. Can you help them figure out what Athena did to get each new set of expressions?

<table>
<thead>
<tr>
<th>Left Expression</th>
<th>Right Expression</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3x + 4 - x - (-2) + x^2$</td>
<td>$-1 + x^2 + 4x - (4 + 2x)$</td>
<td>Original expressions</td>
</tr>
<tr>
<td>$3x + 4 - x - (-2)$</td>
<td>$-1 + 4x - (4 + 2x)$</td>
<td></td>
</tr>
<tr>
<td>$3x + 4 - x + 2$</td>
<td>$-1 + 4x - 4 - 2x$</td>
<td></td>
</tr>
<tr>
<td>$2x + 6$</td>
<td>$2x - 5$</td>
<td></td>
</tr>
<tr>
<td>$6$</td>
<td>$-5$</td>
<td></td>
</tr>
</tbody>
</table>

Because $6 > -5$, the left side is greater.

2-66. For each pair of expressions below, determine which is greater, carefully recording your steps as you go. If you cannot tell which expression is greater, state, “Not enough information.” Make sure that you record your result after each type of simplification. For example, if you flip all of the tiles from the “–” region to the “+” region, record the resulting expression and indicate what you did using either words or symbols. Be ready to share your work with the class.

a. [Diagram]

b. [Diagram]

c. Which is greater: $5 - (2y - 4) - 2$ or $-y - (1 + y) + 4$?

d. Which is greater: $3xy + 9 - 4x - 7 + x$ or $-2x + 3xy - (x - 2)$?
METHODS AND MEANINGS

Associative and Identity Properties

The **Associative Property of Addition** states that when *adding* three or more number or terms together, grouping is not important. That is:

\[(a + b) + c = a + (b + c)\]  \hspace{1cm} \text{For example,} \ (5 + 2) + 6 = 5 + (2 + 6)

The **Associative Property of Multiplication** states that when *multiplying* three or more numbers or terms together, grouping is not important. That is:

\[(a \cdot b) \cdot c = a \cdot (b \cdot c)\]  \hspace{1cm} \text{For example,} \ (5 \cdot 2) \cdot 6 = 5 \cdot (2 \cdot 6)

However, *subtraction* and *division* are not associative, as shown below.

\[5 - (2 - 3) \neq (5 - 2) - 3 \text{ since } 0 \neq 6\]
\[(20 \div 4) \div 2 \neq 20 \div (4 \div 2) \text{ since } 2.5 \neq 10\]

The **Identity Property of Addition** states that adding zero to any expression gives the same expression. That is:

\[a + 0 = a\]  \hspace{1cm} \text{For example,} \ 6 + 0 = 6

The **Identity Property of Multiplication** states that multiplying any expression by one gives the same expression. That is:

\[1 \cdot a = a\]  \hspace{1cm} \text{For example,} \ 1 \cdot 6 = 6
2-67. Sylvia simplified the expressions on the Expression Comparison Mat shown at right. Some of her work is shown. Are all of her moves “legal”? Explain.

2-68. Read the Math Notes boxes in this lesson and for Lesson 2.1.5. Then examine the work below that shows one way to simplify the expression $3x + 8 + (x + 10)$.

Decide which property was used for each step.

<table>
<thead>
<tr>
<th>Expression</th>
<th>What property was used?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3x + 8 + (x + 10)$</td>
<td>Original expression</td>
</tr>
<tr>
<td>$(3x + x + 8) + 10$</td>
<td>a.</td>
</tr>
<tr>
<td>$(4x + 10) + 10$</td>
<td>b.</td>
</tr>
<tr>
<td>$4x + (8 + 10)$</td>
<td>c.</td>
</tr>
<tr>
<td>$4x + 18$</td>
<td>Added like terms</td>
</tr>
</tbody>
</table>

2-69. Examine the tile pattern at right.

a. On graph paper, draw Figures 4 and 5.

b. What would Figure 10 look like? How many tiles would it have? What about Figure 100?

c. Cami has a different tile pattern. She decided to represent the number of tiles of her pattern in a table, as shown below. Can you use the table to predict how many tiles would be in Figure 5 of her tile pattern? How many tiles would Figure 8 have? Explain how you know.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Tiles</td>
<td>5</td>
<td>9</td>
<td>13</td>
<td>17</td>
</tr>
</tbody>
</table>

2-70. Examine the shape made with algebra tiles at right.

a. Write an expression that represents the perimeter of the shape. Then evaluate your expression for $x = 6$ and $y = 10$ units.

b. Write an expression that represents the area of the shape. What is the area if $x = 6$ and $y = 10$ units?

2-71. **CALCULATOR CHECK**

Use your scientific calculator to compute the value of each expression in the left-hand column below. Match each result to an answer in the right-hand column.

a. $-3 + 16 - (-5)$

b. $(3 - 5)(6 + 2)$

c. $17(-23) + 2$

d. $5 - (3 - 17)(-2 + 25)$

e. $-4(-2.25)(-10)$

f. $-1.5 - 2.25 - (-4.5)$

g. $\frac{4 - 3}{2}$

1. $-16$

2. $327$

3. $0.5$

4. $18$

5. $-90$

6. $0.75$

7. $-389$
2.1.8 What if both sides are equal?

Using Algebra Tiles to Solve for $x$

Can you always tell whether one algebraic expression is greater than another? In this section, you will continue to practice the different simplification strategies you have learned so far to compare two expressions and see which one is greater. However, sometimes you do not have enough information about the expressions. When both sides of an equation are equal, you can learn even more about $x$. As you work today, focus on these questions:

How can you simplify?
How can you get $x$ alone?
Is there more than one way to simplify?
Is there always a solution?

2-72. WHICH IS GREATER?

Build each expression represented below with the tiles provided by your teacher. Use “legal” simplification moves to determine which expression is greater, if possible. If it is not possible to determine which expression is greater, explain why it is impossible. Be sure to record your work on your paper.

a. Left

Right

Which is greater?

b. Which is greater:

$x + 1 - (1 - 2x)$ or

$3 + x - 1 - (x - 4)$?
2-73. WHAT IF BOTH SIDES ARE EQUAL?

If the number 5 is compared to the number 7, then it is clear that 7 is greater. However, what if you compare $x$ with 7? In this case, $x$ could be smaller, larger, or equal to 7.

Examine the Expression Comparison Mat below.

a. If the left expression is smaller than the right expression, what does that tell you about the value of $x$?

b. If the left expression is greater than the right expression, what does that tell you about the value of $x$?

c. What if the left expression is equal to the right expression? What does $x$ have to be for the two expressions to be equal?
2-74. SOLVING FOR \( x \)

In later courses, you will learn more about situations like parts (a) and (b) in the preceding problem, called “inequalities.” For now, to learn more about \( x \), assume that the left expression and the right expression are equal. The two expressions will be brought together on one mat to create an Equation Mat, as shown in the figure below. The double line down the center of an Equation Mat represents the word “equals.” It is a wall that separates the left side of an equation from the right side.

a. Obtain the “Equation Mat” resource page from your teacher. Build the equation represented by the Equation Mat at right using algebra tiles. Simplify as much as possible and then solve for \( x \). Be sure to record your work.

b. Build the equation \( 2x - 5 = -1 + 5x + 2 \) using your tiles by placing \( 2x - 5 \) on the left side and \( -1 + 5x + 2 \) on the right side. Then use your simplification skills to simplify this equation as much as possible so that \( x \) is alone on one side of the equation. Use the fact that both sides are equal to solve for \( x \). Record your work.
2-75. Now apply the solving skill from problem 2-74 by building, simplifying, and solving each equation below for $x$. Record your work.

a. $3x - 7 = 2$

b. $1 + 2x - x = x - 5 + x$

c. $3 - 2x = 2x - 5$

d. $3 + 2x - (x + 1) = 3x - 6$

e. $-(x + 3 - x) = 2x - 7$

f. $-4 + 2x + 2 = x + 1 + x$
The Additive Inverse Property states that for every number $a$ there is a number $-a$ such that $a + (-a) = 0$. A common name used for the additive inverse is the opposite. That is, $-a$ is the opposite of $a$. For example, $3 + (-3) = 0$ and $-5 + 5 = 0$.

The Multiplicative Inverse Property states that for every nonzero number $a$ there is a number $\frac{1}{a}$ such that $a \cdot \frac{1}{a} = 1$. A common name used for the multiplicative inverse is the reciprocal. That is, $\frac{1}{a}$ is the reciprocal of $a$. For example, $6 \cdot \frac{1}{6} = 1$. 

METHODS AND MEANINGS

Inverse Properties
2-76. Translate the Equation Mat at right into an equation. Remember that the double line represents “equals.”

2-77. Decide if each statement below is true or false. Explain how you know and name any properties you use. Refer to the Math Notes boxes in this lesson and in Lessons 2.1.5 and 2.1.7 to help you.

a. $-19 + 7 = -7 + 19$

b. $\frac{4}{5} \cdot 1 = \frac{4}{5}$

c. $\frac{3}{2} \cdot \frac{2}{3} = 1$

d. $5 \cdot 7 - 5 \cdot \frac{1}{5} = 34$

2-78. Simplify each expression below as much as possible.

a. $3y - y + 5x + 3 - 7x$

b. $-1 - (-5x) - 2x + 2x^2 + 7$

c. $6x + 2 - 1 - 4x - 3 - 2x + 2$

d. $\frac{2}{3}x - 3y + \frac{1}{3}x + 2y$

2-79. Plot the points $(0, 0)$, $(3, 2)$, and $(6, 4)$ on graph paper. Then draw a line through the points. Name the coordinates of three more points on the same line.

2-80. Evaluate the expressions below for the given values.

a. $6m + 2n^2$ for $m = 7$ and $n = 3$

b. $\frac{5x}{3} - 2$ for $x = -18$

c. $(6x)^2 - \frac{8}{3}$ for $x = 10$

d. $(k - 3)(k + 2)$ for $k = 1$
2.1.9 What is $x$?

More Solving Equations

Today you will explore more equations on the Equation Mat and will examine all of the tools you have developed so far to solve for $x$. While you are working on these problems, be prepared to answer the questions that follow.

How can you simplify?

Can you get the variable alone?

Is there more than one way to simplify?

Is there always a solution?

2-81. On your paper, write the equation represented in each diagram below. For each equation, simplify as much as possible and then solve for $x$ or $y$. Be sure to record your work on your paper.

a. 

b. 

□ = -1
□ = -1
2-82. **IS THERE A SOLUTION?**

While solving homework last night, Richie came across three homework questions that he thinks have no solution. Build each equation below and determine if it has a solution for $x$. If it has a solution, find it. If it does not have a solution, explain why not.

**a.**

```
+  x^2
-  x
```

**b.**

```
+  x
-  x
```

**c.**

```
-  x
```

2-83.  Continue to develop your equation-solving strategies by solving each equation below (if possible). Remember to build each equation, simplify as much as possible, and solve for $x$ or $y$. There are often multiple ways to solve equations, so remember to justify that each step is "legal." If you cannot solve for $x$, explain why not. Be sure to record your work.

a.  $-x + 2 = 4$

b.  $4x - 2 + x = 2x + 8 + 3x$

c.  $4y - 9 + y = 6$

d.  $9 - (2 - 3y) = 6 + 2y - (5 + y)$
2-84. LEARNING LOG

In your Learning Log, explain when you can solve for $x$ in an equation and when you cannot. Be sure to give an example of each situation. Title this entry “Solutions of an Equation” and include today’s date.
**METHODS AND MEANINGS**

**Using an Equation Mat**

An **Equation Mat** can help you visually represent an equation with algebra tiles.

The double line represents the "equal" sign (=).

For each side of the equation, there is a positive and a negative region.

For example, the equation

$$2x - 1 - (-x + 3) = 6 - 2x$$

can be represented by the Equation Mat at right. (Note that there are other possible ways to represent this equation correctly on the Equation Mat.)
2-85. Ling wants to save $87 for tickets to a rock concert. If she has $23 now and will save $4 per week, how long will it take her to get enough money to buy the tickets? Define a variable and then write and solve an equation. (Use the 5-D Process to help you, if needed.) Write your solution as a sentence.

2-86. When you want to convince someone that $8r + (2x - 3)$ is equivalent to $(8r + 2x) - 3$, you can support your claim with the Associative Property of Addition.

Determine if the following statements are true or false. If true, justify your conclusion by stating the appropriate algebraic property. If false, explain how you know.

a. $(81)(38) = (38)(81)$  
   b. $27 + 0 = 27$
   c. $3 - 5 = 5 - 3$  
   d. $19.4 	imes 1 = 19.4$

2-87. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.

a.  
   b.  
   c.  
   d. 

2-88. Ferroza can buy a 24-ounce bag of ferret food for $1.19, or she can buy a 36-ounce bag for $2.89. Which is the better deal? Justify your conclusion.

2-89. This problem is a checkpoint for evaluating expressions and using the Order of Operations. It will be referred to as Checkpoint 2.

Evaluate each expression if $x = -2$, $y = -3$, and $z = 5$.

a. $2x + 3y + z$  
   b. $x - y$  
   c. $2\left(\frac{x + y}{z}\right)$
   d. $3x^2 - 2x + 1$  
   e. $3y(x + x^2 - y)$  
   f. $\frac{-z^2(4 - 2x)}{y-x}$

Check your answers by referring to the Checkpoint 2 materials located at the back of your book.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 2 materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.
Chapter 2 Closure  What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with.

SUMMARIZING MY UNDERSTANDING

This section gives you an opportunity to show what you know about the main math ideas in this chapter.

Simplifying and Solving Equations

Obtain the Chapter 2 Closure Resource Page, Simplifying/ Solving Graphic Organizer (GO), from your teacher.

Your Task:

- Imagine that you are the teacher for some students who have just finished this chapter. You need to make a test to determine how well they know how to use algebra tiles to represent an equation as well as how to simplify and solve for the variable.

- With your team, first brainstorm all of the things that such students would need to know about using algebra tiles. This is the list of the concepts that you would assess with your test.

- Then, as a team, create one equation that would allow you, as the teacher, to assess the students’ understanding of many of the concepts that you listed. Try to make your equation easy enough for the students to do, but challenging enough that you will be able to tell if they understand the list of concepts very well.

- Use the Chapter 2 Closure Graphic Organizer to create an answer key for the problem that you created. Your key should help you assess the work that students turn in. Be sure to include each step as a tile drawing and as a symbolic equation. Also be sure to write an explanation of what the students would do at each step. If your equation takes more than four steps, then continue your work on another piece of paper or on an additional GO page.

- Check your teammates’ answer keys for accuracy.
WHAT HAVE I LEARNED?

This section will help you evaluate which types of problems you have seen with which you feel comfortable and those with which you need more help. This section appears at the end of every chapter to help you check your understanding. Even if your teacher does not assign this section, it is a good idea to try the problems and find out for yourself what you know and what you need to work on.

Solve each problem as completely as you can. The table at the end of this closure section has answers to these problems. It also tells you where you can find additional help and practice on problems like these.
CL 2-90. Examine the Expression Mat at right.

a. Copy the Expression Mat onto your paper.

b. Write an expression for the tiles as they appear.

c. On your drawing, circle all of the zeros that you can find to simplify the expression.

d. Write the completely simplified expression.

CL 2-91. Write expressions for each side of the Expression Comparison Mat. Use “legal” moves to simplify and determine which side is greater.

CL 2-92. Define a variable and then write an equation for the following problem.
Remember that you can use the 5-D Process to help you do this. Then solve your equation and state your answer in a sentence. Show your work in an organized way.

The number of students attending the fall play was 150 more than the number of adults attending. Student tickets cost $3, and adult tickets cost $5. A total of $4730 was collected. How many students attended the play?
CL 2-93. Simplify each expression with or without algebra tiles. Record your steps.

a. \(3 + 7x - (2 + 9x)\)  
b. \(6 - (3x - 4) + 7x - 11\)  
c. \(3x^2 + 10 - y^2 + 4x - 8x^2 - 5y - 8 + y^2 + 3\)

CL 2-94. Copy the pattern at right onto graph paper. Draw Figures 1 and 5 on your paper.

a. How many tiles are in each figure?  
b. How is the pattern changing?  
c. How many tiles would Figure 6 have? Figure 2 Figure 3 Figure 4

CL 2-95. Evaluate \(6x - (3y + 7) - xy\) when \(x = 5\) and \(y = 3\).

CL 2-96. Molly bought 4.25 pounds of fish for $10.20.

a. What is the unit rate (cost per pound)?  
b. What should six pounds of fish cost at the same rate?  
c. Write an equation relating cost (c) with pounds (p).
CL 2-97. For each of the problems above, do the following:

- Draw a bar or number line that represents 0 to 10.

- Color or shade in a portion of the bar that represents your level of understanding and comfort with completing that problem on your own.

If any of your bars are less than a 5, choose one of those problems and complete one of the following tasks:

- Write two questions that you would like to ask about that problem.
- Brainstorm two things that you DO know about that type of problem.

If all of your bars are a 5 or above, choose one of those problems and do one of these tasks:

- Write two questions you might ask or hints you might give to a student who was stuck on the problem.
- Make a new problem that is similar and more challenging than that problem and solve it.
WHAT TOOLS CAN I USE?

You have several tools and references available to help support your learning – your teacher, your study team, your math book, and your Toolkit, to name only a few. At the end of each chapter you will have an opportunity to review your Toolkit for completeness as well as to revise or update it to better reflect your current understanding of big ideas.

The main elements of your Toolkit should be your Learning Logs, Math Notes, and the vocabulary used in this chapter. Math words that are new to this chapter appear in bold in the text. Refer to the lists provided below and follow your teacher's instructions to revise your Toolkit, which will help make it a useful reference for you as you complete this chapter and prepare to begin the next one.

Learning Log Entries
- Lesson 2.1.1 – Algebra Tiles and Variables
- Lesson 2.1.2 – Finding Perimeter and Combining Like Terms
- Lesson 2.1.3 – Meanings of Minus
- Lesson 2.1.3 – Representing Expressions on an Expression Mat
- Lesson 2.1.4 – Using Zeros to Simplify
- Lesson 2.1.5 – Legal Moves for Simplifying and Comparing Expressions
- Lesson 2.1.9 – Solutions of an Equation

Math Notes
- Lesson 2.1.1 – Non-Commensurate
- Lesson 2.1.2 – Mathematics Vocabulary
- Lesson 2.1.3 – Combining Like Terms
- Lesson 2.1.4 – Evaluating Expressions and the Order of Operations
- Lesson 2.1.5 – Commutative Properties
- Lesson 2.1.6 – Simplifying an Expression ("Legal Moves")
- Lesson 2.1.7 – Associative and Identity Properties
- Lesson 2.1.8 – Inverse Properties
- Lesson 2.1.9 – Using an Equation Mat

Mathematical Vocabulary
The following is a list of vocabulary found in this chapter. Some of the words you have been seen in the previous chapter or in previous coursework. The words in bold are the words new to this chapter. Make sure that you are familiar with the terms below and know what they mean. For the words you do not know, refer to the glossary or index. You might also add these words to your Toolkit so that you can reference them in the future.

<table>
<thead>
<tr>
<th>Additive Identity</th>
<th>Additive Inverse</th>
<th>Associative Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>combining like terms</td>
<td>Commutative Property</td>
<td>Equation Mat</td>
</tr>
<tr>
<td>evaluate</td>
<td>Expression (Comparison) Mat</td>
<td></td>
</tr>
<tr>
<td>Multiplicative Identity</td>
<td>Multiplicative Inverse</td>
<td>non-commensurate</td>
</tr>
<tr>
<td>Order of Operations</td>
<td>term</td>
<td>variable</td>
</tr>
</tbody>
</table>
## Answers and Support for Closure Problems

### What Have I Learned?

<table>
<thead>
<tr>
<th>Problem</th>
<th>Solution</th>
<th>Need Help?</th>
<th>More Practice</th>
</tr>
</thead>
</table>
| **CL 2-90.**
  b. 2x - x + 3 - 2 - (x - x + 2 - 1)
  c. One possible answer: x
  d. x
| Lesson 2.1.3 and 2.1.4
MN: 2.1.3 and 2.1.6
LL: 2.1.3 and 2.1.4 | Problems 2-26, 2-38, and 2-42 |
| **CL 2-91.**
  Left: -1 + 2x + 3 - (2x - 2) = 4
  Right: 2x + 2 - x - (2x - x - 2 + 1) = 3
  The left expression is greater than the right expression. |
| Lesson 2.1.5
MN: 2.1.3 and 2.1.6
LL: 2.1.3, 2.1.4, and 2.1.5 | Problems 2-47, 2-56, 2-57, 2-58, 2-63, 2-67, and 2-73 |
| **CL 2-92.**
  Let s represent the number of students, 3s + 5(s - 150) = 4730, s = 685, so 685 students attended the play. |
| MN: 1.1.3 | Problems 2-10, 2-31, and 2-85 |
| **CL 2-93.**
  a. -2x + 1
  b. 4x - 1
  c. -5x^2 + 4x - 5y + 5 |
| Lessons 2.1.1, 2.1.2, and 2.1.5
MN: 2.1.3 and 2.1.6
LL: 2.1.2, 2.1.4, and 2.1.5 | Problems 2-4, 2-17, 2-29, 2-51, 2-69, and 2-78 |
| **CL 2-94.**
  a. |
  b. Each figure has three more tiles than the one before it.
  c. Figure 6 would have 17 tiles. |
| Lesson 1.1.2 | Problems 1-9, 1-10, 1-12, and 2-69 |
| **CL 2-95.**
  6 \cdot 5 - (3 \cdot 3 + 7) - 5 \cdot 3 = -1 |
| Problem 2-9
MN: 2.1.4 | Problems 2-9, 2-19, 2-80, and 2-86 |
| **CL 2-96.**
  a. $2.40
  b. $14.40
  c. c = 2.40p |
| Lessons 1.2.1 and 1.2.2
MN: 1.2.1 and 1.2.2
LL: 1.2.1 | Problems 1-42, 1-43, 1-44, and 1-57 |